

# ORSAY LECTURES ON CONFINEMENT (II)

by

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## The confinement of the heavy quark

In the previous talk [1] we have considered some aspects of the theory of supercharged nuclei, and came to the conclusion, that the superbound atoms are stable mainly due to the Pauli principle.

Before going to the heavy quark, let us discuss briefly, how the problem of the supercharged nucleus can be handled practically. One possibility to formalize it is the following. We have to calculate the Green function of the Dirac equation in external field :

$$\hat{\nabla} G(x, x') = \frac{1}{i} \delta(x - x') \quad , \quad (1)$$

where

$$\hat{\nabla} = \gamma_\mu (\partial_\mu - iA_\mu) \quad . \quad (2)$$

The initial external field is that of the considered nucleus. However, in the presence of a charge  $Z$  the stationary states correspond to atomic states with a charge  $Z - N$  where  $N$  is large enough to fulfil  $Z - N < Z_{cr}$ , i.e. there have to be  $N$  electrons rotating around the nucleus. But if so, we will have to take into account, that the field which acts on each electron is also changing.

The ground state in our case is an atomic type state. This means, that although we don't know what our field is, we can find it. The gradient squared of the zeroth component  $A_0 \equiv A$  of the potential has to be equal

$$\nabla^2 A_0 = e_0^2 (\rho_{nuct} + \rho_{el}) \quad . \quad (3)$$

where  $\rho_{el} = \langle \bar{\psi} \gamma_0 \psi \rangle$ .

The charge density will depend on the potential, and the equation becomes a Thomas-Fermi type equation for the potential existing in this system. If we find a wave function satisfying the equation we will know  $A$ , the Green function and  $\rho$ . This means, that the formal way of solution is just to solve a self-consistent, Thomas-Fermi type equation for the effective atomic potential.

Let us consider the total charge  $Q$

$$Q = \int \rho d^3r \quad ; \quad (4)$$

the density is the average value

$$\rho = \langle \bar{\psi}(r) \psi(r) \rangle = \sum_{\omega < 0} \bar{\psi}_n(r) \psi_n(r) \quad (5)$$

which in the Dirac picture is the sum of all negative energy levels. The total charge  $Q$  will be the sum of the energy levels over all  $\omega_n$ . The sum is divergent, and we have to make a cut-off and subtract the bare particles. This, however, will not be enough. Indeed, if we just subtract the value of the charge of the free vacuum, then at  $Z \neq 0$  the charge of the vacuum (i.e. of the nucleus) will not be equal  $Z$  and will continue to diverge logarithmically. We have to subtract the value at small  $Z$  in a way which gives zero for the total charge of the electron vacuum (this corresponds to the correct renormalization of the charge of the nucleus).

The above procedure does not reflect literally the subtraction of the vacuum charge without external field. This becomes especially obvious, if

one makes use of the Levinson theorem which connects the number of states of a particle in external field in a given energy interval with the phase of scattering of this particle in the same field. The number  $N$  of additional states in the energy interval between  $E_1$  and  $E_2$  is defined by the difference of phase shift

$$N = \frac{1}{\pi} [\delta(E_2) - \delta(E_1)] \quad . \quad (6)$$

Because of this, the number of new states in the Dirac vacuum equals

$$N = \frac{1}{\pi} [\delta(-m) - \delta(-\infty)] \quad . \quad (7)$$

For  $Z < Z_c$  this number should be zero. However, for the Dirac equation in external field this condition is not fulfilled, because  $\delta(-\infty) \neq 0$ , and in order to obtain the proper definition of the vacuum charge we have to subtract a quantity which, generally speaking, depends on  $Z$  ( $\delta(-\infty)$ ). This subtraction means, in fact, that we have to change the interaction with the external field when  $E \rightarrow -\infty$  so that  $\delta(-\infty) = 0$ . ( $\delta(-m)$  can always be considered to be zero if the field is small). If, however,  $Z$  will be increased and becomes more than critical, then, as we saw, the levels will pass the point  $E = -m$  and move to the complex plane. It is easy to check, that every time this happens the phase  $\delta(-m)$  is changing by  $\pi$  and the move of  $n$  levels into the complex plane changes the number of states by  $n$  so that the charge of the vacuum (i.e. of the atom) becomes  $Z - n$ . The value of  $\rho_e$  which enters the equation for the self-consistent field is to be defined by the contribution of levels which passed through  $E = -m$ .

We discussed in detail this concrete structure in order to refer to it in the

following, talking about the heavy quark in the vacuum of the light quark. We shall suppose, that due to gluonic vacuum polarization the effective coupling  $\alpha(r)$  which at small  $r$  has the usual perturbative behaviour, reaches a constant value at  $r \gg r_0$  ( $r_0 \sim 1/\lambda$ ), and this constant value will be more than unity. (Without this ansatz, allowing that  $\alpha$  continues to grow, things are more complicated, but nothing essentially changes).

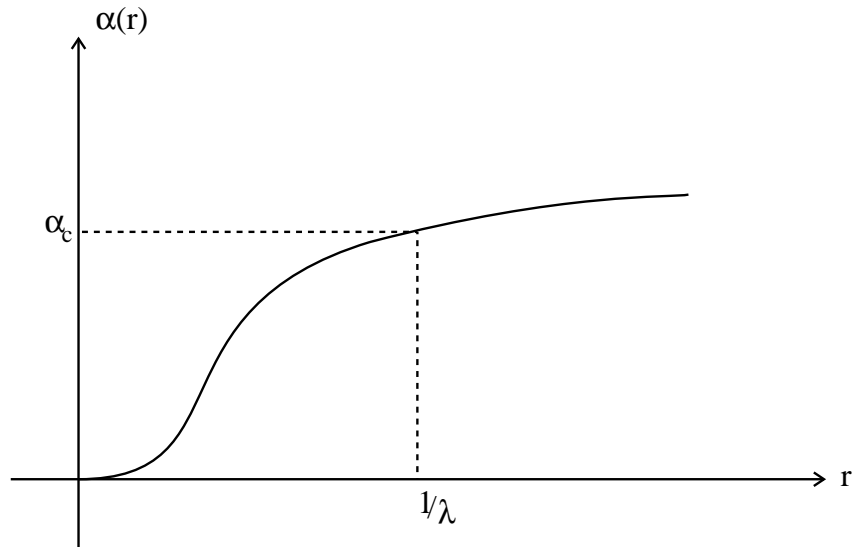
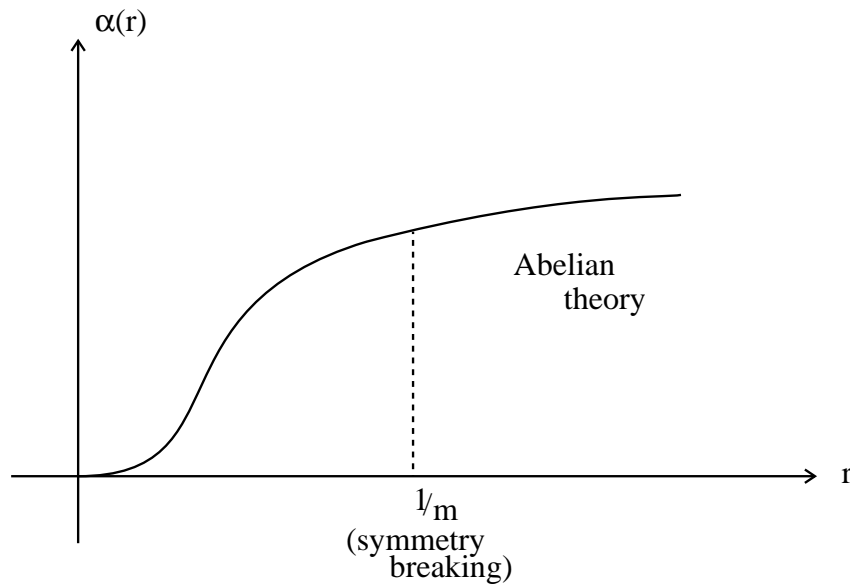


Figure 1

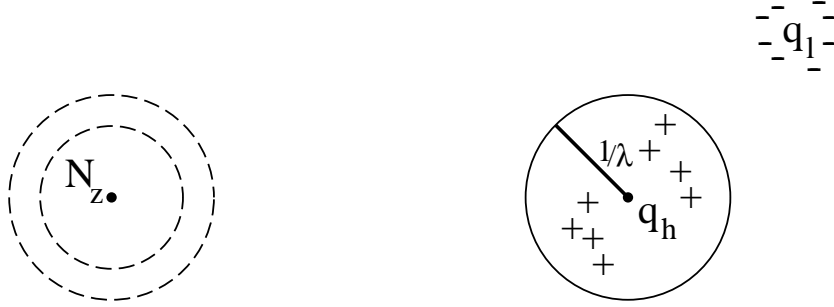
The supposed behaviour of the effective coupling.

It is natural to expect such a behaviour of the charge in QCD. But even an Abelian theory can reveal such a behaviour of charge, if it originates from a non-Abelian theory via spontaneous colour symmetry breaking. In this case the charge will increase, and as a result, 6-7 gluons acquire masses. After that there remain one or two massless objects - “photons” - and the behaviour of charge will be exactly as discussed, because after the symmetry breaking we have  $r_0 = \frac{1}{m}$  (heavy gluon). We can ask now : what happens in the vacuum

of light quarks under these circumstances ? Outside the region where  $\alpha$  is growing, we will have an Abelian theory and we can consider the quark states in the normal way.



If  $\alpha$  becomes more than critical, the corresponding level goes to the Dirac sea, which, consequently, will have to be filled up and we will find an atomic type state. If this is so, the charge density will be exactly the same as in the case of a supercharged nucleus. The  $\lambda_{QCD}$  is analogous to the radius of the nucleus. There is, however, an important difference between QED and this case. Indeed, in QED we consider a nucleus with a charge  $Z$  in the centre, and we put one or two electrons around it to organize an atom. In QCD we have a heavy quark with a very small intrinsic charge.



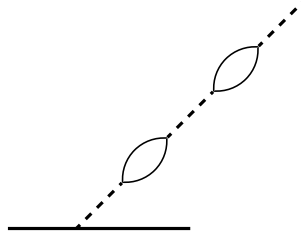
Due to vacuum polarization the charge becomes large inside  $1/\lambda$ . The system creates an empty level, and we have to fill it. This means, that we have to add an intrinsic charge, equal to that of the heavy quark, but with the opposite sign (outside the region of  $1/\lambda$ ). So we have here two intrinsic charges with the total charge zero, and of course the vacuum polarization can never change the total charge. In other words, in QCD the supercritical atom would be a meson-type state with zero colour charge. Our task is to show, that these are not only words - we have to include the mechanism of vacuum polarization formally. In order to do so, let us discuss the problem in a language similar to QED, neglecting non-Abelian fluctuations of the colour field  $A$  - the average field of heavy and light quarks inside the meson. In this case  $A_0$  is defined by the equation

$$\nabla^2 A = e_0^2 (\rho_h - \rho_\ell) \quad , \quad (8)$$

where  $\rho_h$  is the density of the heavy quark, and  $\rho_\ell$  that of the light quark. Again, we can write the equation for the Green function

$$\left( \widehat{\nabla} - m \right) G(x, x') = \frac{1}{i} \delta(x) \quad . \quad (9)$$

But now the problem is, how to calculate this. What means, e. g.,  $\rho_{heavy}$ ? We can not use the expression  $\rho_h = e_0^2 \delta(r)$ , since it does not include the bosonic vacuum polarization, which has to be taken into account. It is well known, that in principle vacuum polarization means summing a diagram like



with gluonic loops inside. The problem is, however, how to write this in normal space-time language. There is a good way to sum this diagram by writing an equation for currents. Knowing the external current and wishing to calculate the total current, we have to solve the equation

$$j_\mu(x) = j_\mu^{ext}(x) + \frac{\alpha_0 b}{2\pi^2} \int \delta'((x-x')^2) j_\mu(x') d^4 x' \quad , \quad (10)$$

which corresponds to the summation of the diagram. We will not derive this equation, because it is almost obvious for the static charge which we are interested in.

Let us consider a static charge, not depending on time. In this case

$$\rho(r) = \rho_\mu^{ext}(r) + \frac{\alpha_0 b}{8\pi^2} \int \frac{d^3 r'}{|r-r'|^3} \rho(r') \quad , \quad |r-r'| > \varepsilon \quad , \quad (11)$$

where  $1/\varepsilon$  is the ultraviolet cut-off,  $b = \frac{11}{3}n_c - \frac{2}{3}n_f$ .

The solution of this equation is very simple and leads to the usual expres-



sion for charge renormalization in QCD. In order to see this, let us introduce the quantity  $Q(r)$  :

$$Q(r) = \int_0^r \rho(r') 4\pi r'^2 dr' \quad . \quad (12)$$

The logarithmic derivative of  $Q(r)$  is

$$\partial_\xi Q(r) = r \frac{\partial Q(r)}{\partial r} = 4\pi r^3 \rho(r) \quad . \quad (13)$$

For  $Q(r)$  we can write the equation

$$\partial_\xi Q(r) = \partial_\xi Q_{ext}(r) + \frac{\alpha_0 b}{8\pi^2} \int_{|r-r'| > \varepsilon} \frac{d^3 r'}{|r-r'|^3} \frac{r^3}{r'^3} \partial_{\xi'} Q(r') \quad . \quad (14)$$

The integration in the right hand-side of this expression contains two logarithmic regions :  $|r-r'| \ll r$  and  $r \gg r'$ . In the first region,  $\partial_{\xi'} Q(r')$  can be substituted by  $\partial_\xi Q(r)$  ; integrating over the second region,  $r'$  in the denominator  $|r-r'|^3$  can be neglected. As a result, we obtain

$$\partial_\xi Q(r) = \partial_\xi Q_{ext}(r) + \frac{\alpha_0 b}{8\pi^2} \int_\varepsilon^r \frac{d^3 r''}{r''^3} \partial_\xi Q(r) + \frac{\alpha_0 b}{2\pi} Q(r) \quad ,$$

which is equivalent to

$$\partial_\xi \left( 1 - \frac{\alpha_0 b}{2\pi} \ln \frac{r}{\varepsilon} \right) Q(r) = \partial_\xi Q_{ext}(r) \quad (15)$$

or

$$Q(r) = \frac{Q_{ext}(r)}{1 - \frac{\alpha_0 b}{2\pi} \ell n \frac{r}{\varepsilon}} \quad . \quad (16)$$

For a point-like charge

$$Q_{ext}(r) = 1 \quad ,$$

and we have

$$A(r) = \frac{\alpha(r)}{r} = \alpha(r) A_{ext} \quad . \quad (17)$$

The concrete expression

$$\alpha(r) = \frac{\alpha_0}{1 - \frac{\alpha_0 b}{2\pi} \ell n \frac{r}{\varepsilon}} \quad (18)$$

obtained from perturbation theory has, of course, an unphysical singularity. For a point-like charge (18) has to be substituted by an expression corresponding to the behaviour of  $\alpha(r)$  as shown in Fig. 1. However, for the distributed charge the relation between the external field and the field which takes into account the polarization is non-local in coordinate space. The correct expression for the relation between the external field and the observable field is local in the momentum space :

$$A(q) = \alpha(q^2) A_{ext}(q) \quad (19)$$

which leads in the coordinate space to an expression of the following type :

$$A(r) = \int K(r - r') A_{ext}(r') d^3 r' \quad , \quad (20)$$

where

$$K(r) = \int e^{iqr} \alpha(q) \frac{d^3 q}{(2\pi)^3} \quad .$$

Similarly,

$$\rho(r) = \int K(r - r') \rho_{ext}(r') d^3 r' \quad . \quad (21)$$

If we now suppose, that  $\alpha(q)$  as a function of  $1/q$  behaves according to Fig. 1, and at large  $q$  values it is defined by perturbation theory, then the equation (8) has to be understood as an equation for  $A_{ext}$  :

$$\nabla^2 A_{ext}(r) = \delta(r) - \bar{\psi}_\ell(r) \gamma_0 \psi_\ell(r) \quad (22)$$

where  $\psi_\ell(r)$  is the solution of the Dirac equation

$$(H + A)\psi = E\psi$$

for the light antiquark in a superbound state in the field  $A(r)$  defined by equation (20). The solution of this problem gives the energy and the features of the meson  $q_n \bar{q}_\ell$  with zero total charge. Due to (21),

$$Q = \int K(r) d^3 r \quad Q_{ext} = \alpha(q=0) Q_{ext} \quad (23)$$

with  $Q = 0$  if  $Q_{ext} = 0$ .

We have just proved, that because of the big charge which appears through vacuum polarization, in the case of QCD the “atomic” bound state

will, indeed, be a meson. The heavy quark will decay into a  $q_h \bar{q}_\ell$ -meson and a light quark :

$$q_h \rightarrow M(q_h \bar{q}_\ell) + q_\ell \quad . \quad (24)$$

In the next lecture we shall consider light quark states. So far there is one important thing to stress, namely if we don't include essential interactions between light quarks in the vacuum, we come to a reasonable conclusion for the case of the heavy quark but, as we will just see, to an unreasonable one about the light quark. Indeed, let us try to extend the considered procedure to the latter case. Suppose, that there is a light quark moving, and a potential acts on its vacuum. What will we see classically ? Since the Coulomb field is a vector field, it is shrinking, but the total integral remains the same. Because of this, we will find immediately, that there is a bound state in this potential even for fastly moving particles. This, however, means, that we have here an unstable state, which has to be filled, and as a consequence the light quark will decay into a meson and a light quark again :

$$q_\ell \rightarrow M + q_\ell$$

which, of course, contradicts the energy conservation, unless the appearing meson is of negative energy. In order to have a self-consistent picture, we have to suppose that the light quark in the vacuum will interact so strongly that there have to be negative energy levels and the whole vacuum has to be rearranged. So from the picture we described we come quite naturally to light quark interactions. We will see, that these interactions are, indeed, very strong and lead to the confinement of light quarks which will take place at

relatively small  $\frac{\alpha}{\pi}$  values ; this means, that the overall corrections for vacuum polarization will not be large.

This is for the future. What we have to add now, is, that even in the language which was accepted so far, with no strong interactions between particles in the vacuum, the problem in real QCD which is non-Abelian is more complicated. In QED we have one charge, and all the electrons in the vacuum interact with this charge, independently from each other. In QCD this can take place only, if the field of the heavy quark is an Abelian one. It is highly probable, that this is, indeed, the case, when the field of the heavy quark becomes large as a result of gluonic vacuum polarization.

**Reference:**

[1] V.N. Gribov, Orsay Lectures on confinement (I)

The theory of supercharged nucleus, LPTHE ORSAY 92/60 (1993).